1.Moment-Based Feature

- Many shape feature can be represented in terms of moments.
- Suppose we have any shape and that particular shape is represented by a region $\mathcal{R}$ that containing N pixels, we have the following:

1. Center of mass
2. Orientation
3. Bounding rectangle
4. Best-fit ellipse
5. Eccentricity

1.Center of mass

- $\bar{\boldsymbol{m}}=\frac{1}{N}\sum_{(m,n)\in\mathcal{R}}\sum m,$

  $\bar{\boldsymbol{n}}=\frac{1}{N}\sum_{(m,n)\in\mathcal{R}}\sum n,$

- The (p,q)order central moments become

  $\boldsymbol{\mu}_{\mathbf{p,q}}=\sum_{(\boldsymbol{m},\boldsymbol{n})\in\boldsymbol{\mathcal{R}}}\sum(\mathbf{m}-\bar{\boldsymbol{m}})^{p}(\mathbf{n}-\bar{\boldsymbol{n}})^{\mathbf{q}}$

  where m , n denotes the pixel value.

  $\mathcal{R}$ represents the region.

  $\boldsymbol{\mu}$ represents the central moments

  N represents total no. of pixel that any shape is containing.

2.Orientation

- Orientation is defined as the angle of axis of the least moment of inertia.
- It is obtained by minimizing with respect to θ the sum is:
-

$$\boldsymbol{I}(\theta)=\sum\sum_{(m,n)\in\mathcal{R}} D^2(m,n)=$$

$$\sum\sum_{(m,n)\in\mathcal{R}} [(\boldsymbol{n}-\overline{\boldsymbol{n}})\cos\boldsymbol{\theta}-(\boldsymbol{m}-\overline{\boldsymbol{m}})\sin\boldsymbol{\theta}]^2$$

- The result is $\boldsymbol{\theta}=\frac{1}{2}\tan^{-1}\left[\frac{2\mu_{1,1}}{\mu_{2,0}-\mu_{0,2}}\right]$

Cond...

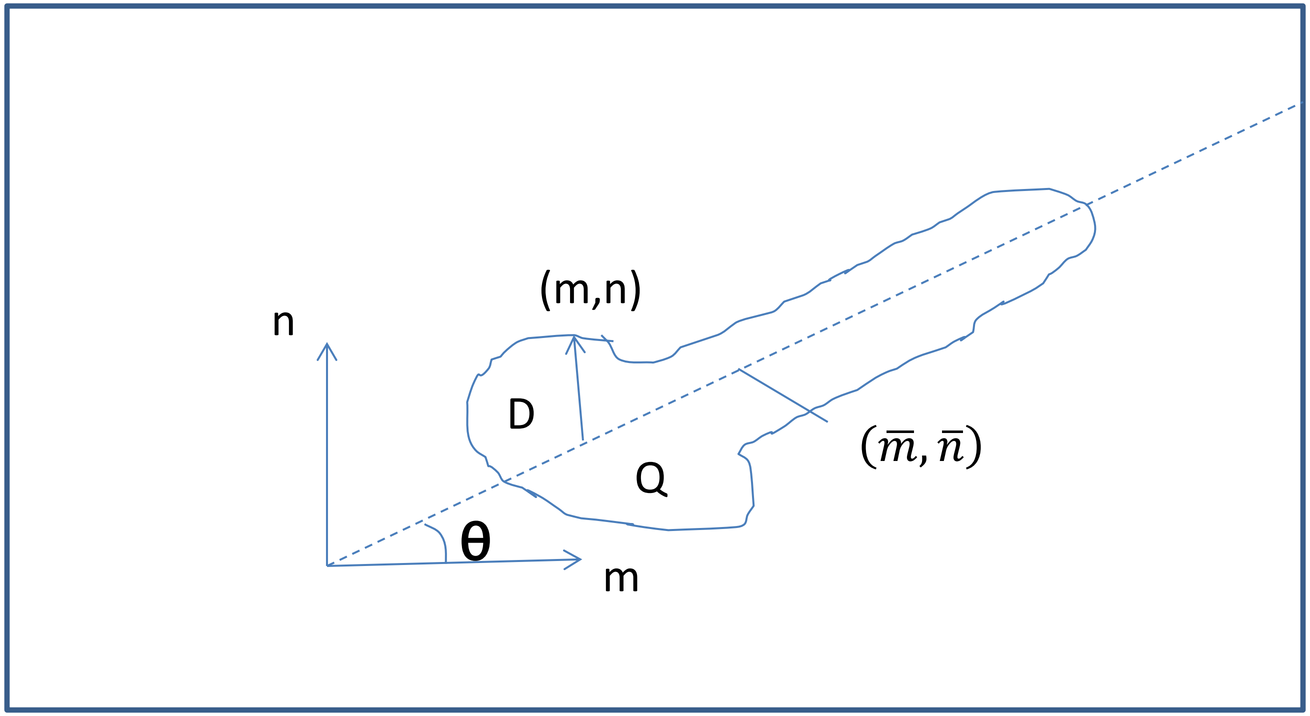
Where

m , n denotes the pixel value.

$\mathcal{R}$ represents the region.

$\boldsymbol{\mu}$ represents the central moments.

Orientation

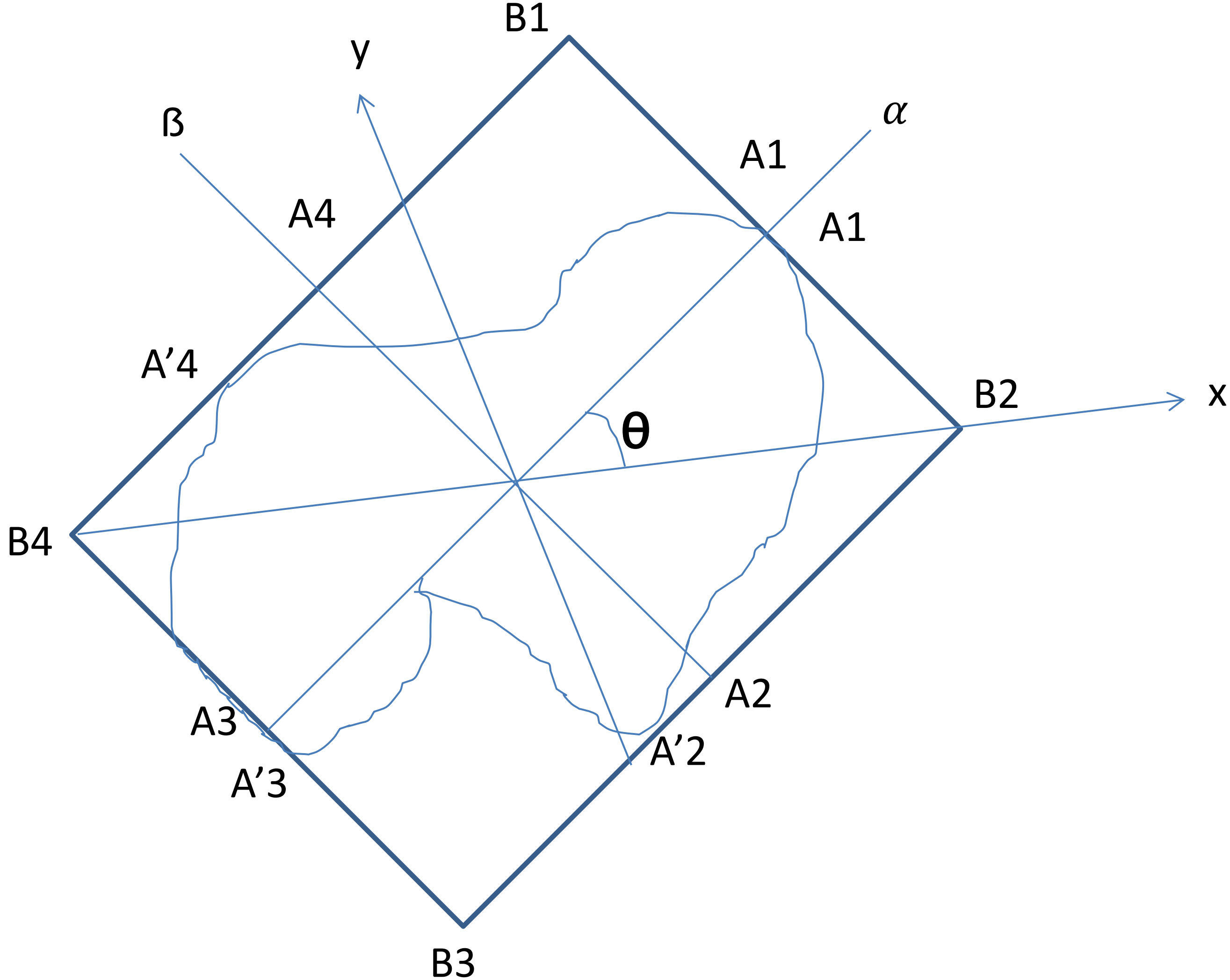


3.Bounding Rectangle

- The bounding rectangle is the smallest rectangle enclosing the object that is also aligned with its orientation.

- Once θ is known we use the transformation
  $\boldsymbol{\alpha = x\cos\theta + y\sin\theta}$
  $\boldsymbol{\beta = -x\sin\theta + y\cos\theta}$ on the boundary points and search for $\alpha_{\text{min}}, \alpha_{\text{max}}, \beta_{\text{min}}, \beta_{\text{max}}$.

Boundary Rectangle



4.Best-fit ellipse

- The best fit ellipse is the ellipse whose second moment equals that of the object.
- Suppose we have any best fit ellipse and a and b denote the lengths of semi-major and semi-minor,respectively, of the best fit ellipse.

- The least and the greatest moments of inertia for an ellipse are

$$I_{\text{min}} = \frac{\pi}{4} ab^3$$

$$I_{\text{max}} = \frac{\pi}{4} a^3 b$$

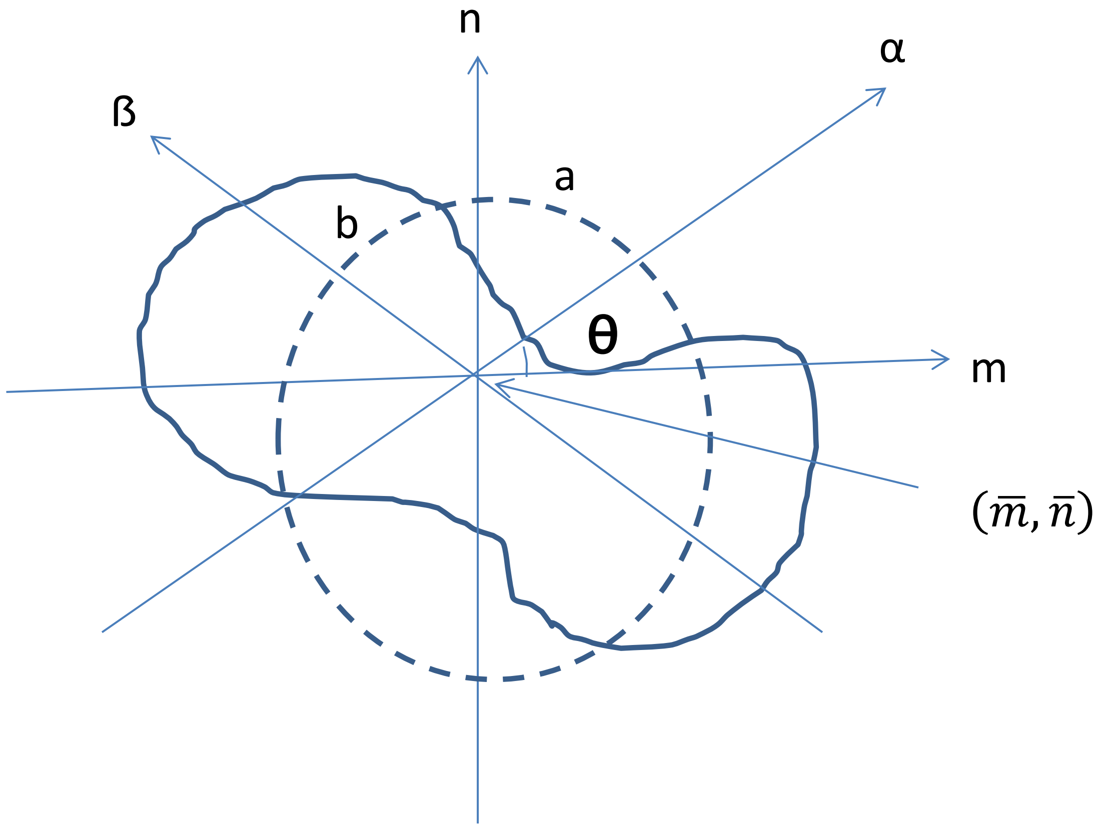
where a=length of semi major axis.

b=length of semi-minor axis.

I_{min} **=least moment of inertia.**

I_{max}**=greatest moment of inertia.**

Best Fit Eclipse



Cond....

- Suppose we have any orientation θ,then the moment can be calculated as

$$I'_{min} = \sum\sum_{(m,n)\in\mathcal{R}} [(n-\bar{n})\cos\theta - (m-\bar{m})\sin\theta]$$

$$I'_{max} = \sum\sum_{(m,n)\in\mathcal{R}} [(n-\bar{n})\sin\theta - (m-\bar{m})\cos\theta]^2$$

- For the best-fit ellipse it is required to have
$I_{min} = I'_{min}$ & $I_{max} = I'_{max}$,which gives

$$a = \left(\frac{4}{\pi}\right)^{1/4} \left[\frac{(I'_{max})^3}{I'_{min}}\right]^{1/8}$$

$$b = \left(\frac{4}{\pi}\right)^{1/4} \left[\frac{(I'_{min})^3}{I'_{max}}\right]^{1/8}$$

5.Eccentricity

- Generally represented by ε.
- $\boldsymbol{\varepsilon} \triangleq \frac{(\mu_{2,0} - \mu_{0,2})^2 + 4\,\mu_{1,1}}{\boldsymbol{area}}$
- Some more representations of eccentricity are $\boldsymbol{R_{max}/R_{min}}$, $\boldsymbol{I'_{max}/I'_{min}}$ and $^{\boldsymbol{a}}/_{\boldsymbol{b}}$.

  where $\boldsymbol{\mu}$ represents the central moments

References

1. Anil K. Jain, “Fundamentals of Digital Image Processing", PHI Learning Education ,Inc.,3rd Edition

Thank You!